QUIZ 9 - SOLUTIONS

1. Find the volume of the solid of revolution formed by rotating about the $x$-axis each region bounded by the following curves.

   \[ y = \frac{1}{2}x + 4, \quad y = 0, \quad x = 0, \quad x = 5 \]

Answer:

\[
\text{Volume} = \int_0^5 \pi \left( \frac{1}{2}x + 4 \right)^2 \, dx = \int_0^5 \frac{\pi}{4} x^2 + 4\pi x + 16\pi \, dx \\
= \left. \frac{\pi}{12} x^3 + 2\pi x^2 + 16\pi x \right|_0^5 \\
= \frac{125\pi}{12} + 50\pi + 80\pi \\
= \frac{125\pi}{12} + 130\pi
\]

2. Find the average value of the function \( y = (2x + 1)^{1/2} \) on the interval \([1, 13]\).

Answer:

\[
f_{\text{Ave}} = \frac{1}{13 - 1} \int_1^{13} (2x + 1)^{1/2} \, dx \\
\text{We do a } u \text{ substitution with } u = 2x + 1 \text{ and } du = 2 \, dx. \text{ So}
\]

\[
f_{\text{Ave}} = \frac{1}{12} \cdot \frac{1}{2} \int_3^{27} u^{1/2} \, du \\
= \frac{1}{24} \cdot \frac{u^{3/2}}{3/2} \bigg|_3^{27} \\
= \frac{1}{36} \left( 27^{3/2} - 3^{3/2} \right) \\
\approx 3.75277674973257
\]
QUIZ 10 - SOLUTIONS

1. Find the volume of the solid of revolution formed by rotating about the $x$-axis each region bounded by the following curves.

\[ y = \ln x, \quad y = 0, \quad x = 1, \quad x = e \]

Answer:

Volume = \( \int_1^e \pi (\ln x)^2 \, dx \)

We integrate by parts, with \( u = \pi (\ln x)^2 \) and \( dv = dx \), then \( du = 2\pi (\ln x) \frac{1}{x} \, dx \) and \( v = x \).

Volume = \( \int_1^e \pi (\ln x)^2 \, dx \)

\[ = \pi (\ln x)^2 x \bigg|_1^e - \int_1^e 2\pi (\ln x) \frac{1}{x} \, dx \]

\[ = \pi (\ln x)^2 x \bigg|_1^e - \int_1^e 2\pi \ln x \, dx \]

We integrate by parts again, \( u = 2\pi \ln x \) and \( dv = dx \), then \( du = 2\pi \frac{1}{x} \, dx \) and \( v = x \).

Volume = \( \pi (\ln x)^2 x \bigg|_1^e - \int_1^e 2\pi \ln x \, dx \)

\[ = (\pi \ln e)^2 e - \pi (\ln 1)^2 1 - \left( x2\pi \ln x \bigg|_1^e - \int_1^e x2\pi \frac{1}{x} \, dx \right) \]

\[ = e\pi - ((e2\pi \ln e - 1 \cdot 2\pi \ln 1) - (2\pi x \bigg|_1^e)) \]

\[ = e\pi - (2e\pi - 2\pi) \]

\[ = e\pi - 2\pi \]
2. Find
\[ \int \frac{2}{x^\pi} \, dx \]

Answer:
\[
\int_2^\infty \frac{2}{x^\pi} \, dx = \lim_{b \to \infty} \int_2^b \frac{2}{x^\pi} \, dx \\
= \lim_{b \to \infty} \left[ \frac{2}{(1 - \pi)x^{\pi-1}} \right]_2^b \\
= \lim_{b \to \infty} \frac{2}{(1 - \pi)b^{\pi-1}} - \frac{2}{(1 - \pi)2^{\pi-1}} \\
= \frac{1}{(\pi - 1)2^{\pi-2}}
\]