EXAM 3

Please show all your work.

1. 
   \[ f(x, y) = \frac{3x^2y^3}{x^2 + y^2} \]
   a. (5 points) Find \( f_x(2, -1) \).
   
   b. (5 points) Find \( f_y(-4, 3) \).
2. 

\[ f(x, y) = x \sin(\pi y) \]

a. (5 points) Find \( f_x(2, -1) \).

b. (5 points) Find \( f_y(-4, 3) \).
3. (10 points) Evaluate

\[ \int_{1}^{3} \int_{0}^{2} x(x^2 + y) \, dx \, dy \]
4. (10 points) Integrate the function $f(x, y) = x + y$ over the region defined by the curves $y = x^2$ and $y = 2x + 3$. 
5. (10 points) Solve for $y$.

$$3x^2 - 2 \frac{dy}{dx} = 2$$
6. (10 points) Solve for $y$ with the initial value $y = 1$ and $x = 0$.

$$(2x + 3)y = \frac{dy}{dx}$$
7. (10 points) Solve for $y$ with the initial value $y(4) = 50$.

$$x \frac{dy}{dx} + xy = 3 - y$$
8. (10 points) Solve for $y$ with the initial value $y(2) = 12$.

$$x \frac{dy}{dx} = x^2 - 5y$$
9. A tank holds 200 gallons of water that contains 20 pounds of dissolved salt. A brine (salt) solution is flowing into the tank at the rate of 3 gallons per minute, while the solution flows out of the tank at the same rate. The brine solution entering the tank has a salt concentration of 2 pounds per gallon.

a. (6 points) Find an expression for the amount of salt in the tank at any time.

b. (2 points) How much salt is present after 2 hours?

c. (2 points) As time increases, what happens to the salt concentration?
10. All parts of this problem concern the differential equation
\[
\frac{dy}{dx} = 2x^2y
\]
a. (7 points) Draw a vector field for this differential equation. Use the integer points in \(-2 \leq x \leq 2\) and \(-2 \leq y \leq 2\). Include the slopes in your picture.
b. (7 points) Use Euler’s method to approximate $y(2.5)$ with initial value $x = 1$, $y = 1$ and $h = \frac{1}{2}$. 
c. (6 points) Solve the differential equation exactly with initial value $x = 1, y = 1$. Find $y(2.5)$. 