1. Determine if the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n + 5}{\sqrt[3]{n^7} + n^2} \]

We will use the Limit Comparison Test to compare this series to

\[ \sum_{n=1}^{\infty} \frac{n}{n^{3/4}} = \sum_{n=1}^{\infty} \frac{1}{n^{4/3}} \]

which converges as it is a \( p \)-series with \( p = \frac{4}{3} > 1 \).

\[
\lim_{n \to \infty} \frac{n + 5}{\sqrt[3]{n^7} + n^2} = \lim_{n \to \infty} \frac{n + 5}{n} \cdot \frac{1}{\sqrt[3]{n^7} + n^2} \\
= \lim_{n \to \infty} (1 + \frac{5}{n}) \cdot \frac{1}{\sqrt[3]{n^7} + n^2} \\
= \lim_{n \to \infty} (1 + \frac{5}{n}) \cdot \frac{1}{\sqrt[3]{1 + \frac{1}{n^3}}} \\
= 1.
\]

As the limit is converging to a number which is not zero, we may apply the Limit Comparison Test to determine that the series converges.

2. Determine if the series converges or diverges.

\[ \sum_{n=2}^{\infty} (-1)^{n+1} \frac{n}{\ln(n)} \]

We will show that this diverges by the Test for Divergence.

\[
\lim_{n \to \infty} \frac{n}{\ln(n)} = \lim_{x \to \infty} \frac{x}{\ln(x)} \\
= \lim_{x \to \infty} \frac{1}{\ln(x)} \\
= \lim_{x \to \infty} \frac{1}{x} \\
= \lim_{x \to \infty} x = \infty
\]

Since \( \lim_{n \to \infty} \frac{n}{\ln(n)} \) diverges, \( \lim_{n \to \infty} (-1)^{n+1} \frac{n}{\ln(n)} \) does not converge to 0. Therefore by the Test for Divergence the series diverges.

† l’Hôpital’s Rule